

Character Table for D_5

$$D_5 = \langle r, \sigma \rangle, \quad o(r) = 5, \quad o(\sigma) = 2, \quad r\sigma = \sigma r^{-1}$$

$$H = \langle r \rangle \cong \mathbb{Z}_5$$

	e	r	r ²	r ³	r ⁴
χ_1	1	1	1	1	1
χ_2	1	ζ	ζ^2	ζ^3	ζ^4
χ_3	1	ζ^2	ζ^4	ζ	ζ^3
χ_4	1	ζ^3	ζ	ζ^4	ζ^2
χ_5	1	ζ^4	ζ^3	ζ^2	ζ

$$\zeta = e^{2\pi i/5}$$

D_5	e	r	r ²	σ
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Ind χ_1	2	2	2	0
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Ind χ_2	2	α	β	0
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Ind χ_3	2	β	α	0
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Ind χ_4	2	β	α	0
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Ind χ_5	2	α	β	0
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$$\alpha = \zeta + \zeta^{-1} \in \mathbb{R}$$

$$\beta = \zeta^2 + \zeta^{-2} \in \mathbb{R}$$

$$\langle \text{Ind } \chi_k, \text{Ind } \chi_t \rangle = 1$$

if $k=2,3,4,5$

D_5	1 e	2 r	2 r^2	5 j
x_1	1	1	1	1
x_2	1	1	1	-1
x_3	2	α	β	0
x_4	2	β	α	0
	10	5	5	2